



## Liquidity Black Holes<sup>\*</sup>

STEPHEN MORRIS<sup>1</sup> and HYUN SONG SHIN<sup>2</sup>

<sup>1</sup>*Cowles Foundation, Yale University;* <sup>2</sup>*London School of Economics*

**Abstract.** Traders with short horizons and privately known loss limits interact in a market for a risky asset. Risk-averse, long horizon traders generate a downward sloping residual demand curve that faces the short-horizon traders. When the price falls close to the loss limits of the short horizon traders, selling of the risky asset by any trader increases the incentives for others to sell. Sales become mutually reinforcing among the short term traders, and payoffs analogous to a bank run are generated. A “liquidity black hole” is the analogue of the run outcome in a bank run model. Short horizon traders sell because others sell. Using global game techniques, we solve for the unique trigger point at which the liquidity black hole comes into existence. Empirical implications include the sharp V-shaped pattern in prices around the time of the liquidity black hole.

### 1. Introduction

Occasionally, financial markets experience episodes of turbulence of such an extreme kind that they appear to stop functioning. Such episodes are marked by a heavily one-sided order flow, rapid price changes, and financial distress on the part of many of the traders. The 1987 stock market crash is perhaps the most glaring example of such an episode, but there are other, more recent examples such as the collapse of the dollar against the yen on 7 October 1998, and instances of distressed trading in some fixed income markets during the LTCM crisis in the summer of 1998. Practitioners dub such episodes “liquidity holes” or, more dramatically, “liquidity black holes” (Taleb, 1997, pp. 68–69; Persaud, 2001).

Liquidity black holes are not simply instances of large price changes. Public announcements of important macroeconomic statistics, such as the U.S. employment report or GDP growth estimates, are sometimes marked by large, discrete price changes at the time of announcement. However, such price changes are arguably the signs of a smoothly functioning market that is able to incorporate new information quickly. The market typically finds composure quite rapidly after such discrete price changes, as shown by Fleming and Remolona (1999) for the US Treasury securities market.

In contrast, liquidity black holes have the feature that they seem to gather momentum from the endogenous responses of the market participants themselves.

---

<sup>\*</sup> This is a much revised version of an earlier paper circulated under the title ‘Market Risk with Interdependent Choice’. We thank Franklin Allen, Brandon Davies, Marco Pagano, Carol Osler, Guillaume Plantin, Jon Danielsson, Amil Dasgupta, Markus Brunnermeier, Jean-Pierre Zigrand and Frank Heinemann for comments at various stages of this project.

Rather like a tropical storm, they appear to gather more energy as they develop. Part of the explanation for the endogenous feedback mechanism lies in the idea that the incentives facing traders undergo changes when prices change. Market distress can feed on itself. When asset prices fall, some traders may get close to their loss limits and are induced to sell. But this selling pressure sets off further downward pressure on asset prices, which induces a further round of selling, and so on. Portfolio insurance based on dynamic hedging rules is perhaps the best-known example of such feedback, but similar forces will operate whenever traders face constraints on their behaviour that shorten their decision horizons. Daily loss limits and other controls on traders' discretion arise as a response to agency problems within a financial institution, and are there for good reason. However, they have the effect of shortening the decision horizons of the traders.

In what follows, we study traders with short decision horizons who have exogenously given loss limits. Their short decision horizon arises from the threat that a breach of the loss limit results in dismissal – a bad outcome for the trader. However, the loss limit of each trader is private information to that trader. Also, although the loss limits across traders can differ, they are correlated, *ex ante*, reflecting broadly similar risk factors to their portfolios. More concretely, we suppose that the traders interact in a market for a risky asset, where risk-averse, long horizon traders generate a downward sloping residual demand curve. When the price falls close to the loss limits of the short horizon traders, selling of the risky asset by any trader increases the incentives for others to sell. This is because sales tend to drive down the market-clearing price, and the probability of breaching one's own loss limit increases. This sharpens the incentives for other traders to sell. In this way, sales become reinforcing between the short term traders. In particular, the payoffs facing the short horizon traders are analogous to a bank run game. A "liquidity black hole" is the analogue of the run outcome in a bank run model. Short horizon traders sell because others sell.

If the loss limits were common knowledge, the payoffs have the potential to generate multiple equilibria. Traders sell if they believe others sell, but if they believe that others will hold their nerve and not sell, they will refrain from selling. Such multiplicity of equilibria is a well-known feature of the bank run model of Diamond and Dybvig (1983). However, when loss limits are not common knowledge, as is more reasonable, the global game techniques of Morris and Shin (1998, 2003) and Goldstein and Pauzner (2000) can be employed to solve for the unique trigger point at which the liquidity black hole comes into existence.<sup>1</sup> Our approach is game-theoretic, relying on explicit analysis of the payoff interactions between strategic traders. In this respect, we depart from the more familiar analyses of crashes that rely on competitive, price-taking behaviour, such as in Genotte and Leland (1990).

---

<sup>1</sup> Global game techniques have been in use in economics for some time, but they are less well established in the finance literature. Some exceptions include Plantin (2003) and Bruche (2002). Abreu and Brunnermeier (2003) employ techniques closely related to global games.

Our model is consistent with a stylised taxonomy of institutions operating in modern securities markets in terms of two broad classes of financial institutions.<sup>2</sup> Type A institutions are mainly the large banks – tightly regulated financial entities with large amounts of surplus capital, often underpinned by retail deposits. They are primarily investors in long term illiquid assets (corporate and retail loans, including mortgages), but will hold securities if the return is attractive enough – that is, offering a return commensurate with that of their illiquid assets. They account for their profit and loss on an accrual basis.

In contrast, there are the type B institutions, such as hedge funds. These are highly illiquid when viewed from their funding base. They have relatively little capital and are lightly regulated. They are solely traders in securities, and secure their funding primarily by pledging their assets in the overnight repo market (mainly to the type A institutions). Type B institutions account for profit and loss on a mark to market basis, and have rigorous internal controls based on their daily profit and loss and daily liquidity positions.

Any exogenous shock to the prices of securities held by type B institutions can produce a significant liquidity problem, as their supply of daily liquidity may be curtailed both by falling asset prices and by more stringent lending by type A institutions who worry about the credit risk of type B institutions. In response, type B institutions will attempt to shore up their business by curtailing their activities and by selling assets. The buyers of these assets will typically be type A institutions, but they will only buy when there is a significant discount over fair value, so that they can hold the asset to maturity and make a return commensurate with their illiquid assets. They treat the assets as illiquid and account for them on an accrual basis.

Our model follows this taxonomy closely. Type A institutions are embodied in the risk-averse long horizon traders who generate the downward-sloping residual demand curve. The active, risk-neutral traders can be seen as the type B institutions.

The idea that the residual demand curve facing active traders is not infinitely elastic is a familiar one. It is an important cornerstone of the market microstructure literature. It has also figured in explanations of market distress. Following the 1987 stock market crash, Grossman and Miller (1988) proposed a story for the crash by positing a role for risk-averse market makers who accommodate order flows and are compensated with higher expected return. Campbell et al. (1993) find evidence consistent with this hypothesis by showing that returns accompanied by high volume tend to be reversed more strongly. Pastor and Stambaugh (2002) provide further evidence for this hypothesis by finding a role for a liquidity factor in an empirical asset pricing model, based on the idea that price reversals often follow liquidity shortages. Bernardo and Welch (2001) and Brunnermeier and Pedersen (2002) have used this device in modelling limited liquidity facing active traders.<sup>3</sup>

---

<sup>2</sup> We are indebted to Brandon Davies of Barclays Group for this taxonomy.

<sup>3</sup> Lustig (2001) emphasizes solvency constraints in giving rise to a liquidity-risk factor in addition to aggregate consumption risk. Acharya and Pedersen (2002) develop a model in which each asset's

More generally, the limited capacity of the market to absorb sales of assets has figured prominently in the literature on banking and financial crises (see Allen and Gale, 2001; Gorton and Huang, 2003 and Schnabel and Shin 2002), where the price repercussions of asset sales have important adverse welfare consequences. Similarly, the inefficient liquidation of long assets in Diamond and Rajan (2000) has an analogous effect. The shortage of aggregate liquidity that such liquidations bring about can generate contagious failures in the banking system.

Some market microstructure studies show evidence consistent with an endogenous trading response that magnifies the initial price change. Cohen and Shin (2001) show that the US Treasury securities market exhibit evidence of positive feedback trading during periods of rapid price changes and heavy order flow. Indeed, even for macroeconomic announcements, Evans and Lyons (2003) find that the foreign exchange market relies on the order flow of the traders in order to interpret the significance of the macro announcement. Hasbrouck (2000) finds that a flow of new market orders for a stock are accompanied by the withdrawal of limit orders on the opposite side. Danielsson and Payne's (2001) study of foreign exchange trading on the Reuters 2000 trading platform shows how the demand or supply curve disappears from the market when the price is moving against it, only to reappear when the market has regained composure. Osler (2002) studies the effect of stop-loss orders on the dynamics of prices in the foreign exchange market, and finds that stop loss orders have a bigger impact on prices, which also last longer than other types of orders. The interpretation that emerges from these studies is that smaller versions of such liquidity gaps are pervasive in active markets – that the market undergoes many “mini liquidity gaps” several times per day.

The next section presents the model. We then proceed to solve for the equilibrium in the trading game using global game techniques. We conclude with a discussion of the empirical implications and the endogenous nature of market risk.

## 2. Model

An asset is traded at two consecutive dates, and then is liquidated. We index the two trading dates by 1 and 2. The liquidation value of the asset at date 2 when viewed from date 0 is given by

$$v + z \tag{2.1}$$

where  $v$  and  $z$  are two independent random variables.  $z$  is normally distributed with mean zero and variance  $\sigma^2$ , and is realized at date 2. The realization of  $v$  is return is net of a stochastic liquidity cost, and expected returns are related to return covariances with the aggregate liquidity cost (as well as to three other covariances). Gromb and Vayanos (2002) build on the intuitions of Shleifer and Vishny (1997) and show that margin constraints have a similar effect in limiting the ability of arbitrageurs to exploit price differences. Holmström and Tirole (2001) propose a role for a related notion of liquidity arising from the limited pledgeability of assets held by firms due to agency problems. Kyle and Xiong (2001) propose a mechanism for contagion arising from wealth effects.

common knowledge at date 1. We do not need to impose any assumptions on the distribution from which  $v$  is drawn. The important feature for our exercise is that, at date 1 (after the realization of  $v$ ), it is common knowledge that the liquidation value of the asset is normal with mean  $v$  and variance  $\sigma^2$ .

There are two groups of traders in the market, and the realization of  $v$  at date 1 is common knowledge among all of them. There is, first, a continuum of risk neutral traders of measure 1. These are the “type B” traders alluded to in the introduction. Each trader holds 1 unit of the asset. We may think of them as proprietary traders (e.g. at an investment bank or hedge fund). They are subject to an incentive contract in which their payoff is proportional to the final liquidation value of the asset. However, these traders are also subject to a loss limit at date 1, as will be described in more detail below. If a trader’s loss between dates 0 and 1 exceeds this limit, then the trader is dismissed. Dismissal is a bad outcome for the trader, and the trader’s decision reflects the tradeoff between keeping his trading position open (and reaping the rewards if the liquidation value of the asset is high), against the risk of dismissal at date 1 if his loss limit is breached at date 1. We do not model explicitly the agency problems that motivate the loss limit. The loss limit is taken to be exogenous for our purpose.

Alongside this group of risk-neutral traders is a risk-averse market-making sector of the economy. They are the “type A” traders alluded to above. The market-making sector provides the residual demand curve facing the risk-neutral traders as a whole, in the manner envisaged by Grossman and Miller (1988) and Campbell et al. (1993).

We represent the market-making sector by means of a representative trader with constant absolute risk aversion  $\gamma$  who posts limit buy orders for the asset at date 1 that coincides with his competitive demand curve. At date 1 (when  $v$  is common knowledge), the liquidation value of the asset is commonly known to be normally distributed with mean  $v$  and variance  $\sigma^2$ . From the linearity of demand with Gaussian uncertainty and exponential utility, the market-making sector’s limit orders define the linear residual demand curve:

$$d = \frac{v - p}{\gamma \sigma^2}$$

where  $p$  is the price of the asset at date 1. Thus, if the aggregate net supply of the asset from the risk-neutral traders is  $s$ , price at date 1 satisfies

$$p = v - cs \tag{2.2}$$

where  $c$  is the constant  $\gamma \sigma^2$ . Since the market-making sector is risk-averse, it must be compensated for taking over the risky asset at date 1, so that the price of the asset falls short of its expected payoff by the amount  $cs$ .

## 2.1. LOSS LIMITS

In the absence of any artificial impediments, the efficient allocation is for the risk-neutral traders to hold all of the risky asset. However, the risk-neutral traders are subject to a loss limit that constrains their actions. The loss limit is a trigger price or “stop price”  $q_i$  for trader  $i$  such that if

$$p < q_i$$

then trader  $i$  is dismissed at date 1. Dismissal is a bad outcome for the trader, and results in a payoff of 0. The loss limits of the traders should be construed as being determined in part by the overall risk position and portfolio composition of their employers. Loss limits therefore differ across traders, and information regarding such limits are valuable for rival traders who want to second guess the trader’s actions. In general, the loss limits will fail to be common knowledge among the traders. This will be the crucial feature of our model that drives the main results. We will also assume that, conditional on being dismissed, the trader prefers to maximize the value of his trading book. The idea here is that the trader is treated more leniently if the loss is smaller.

We will model the loss limits as random variables that are closely correlated across the traders. The similarity of the loss limits is motivated by the similar risk factors and trading positions of active, type B traders. We will impose a joint density over loss limits that arises from an additive structure in which there is a common component and an additive idiosyncratic component. Trader  $i$ ’s loss limit  $q_i$  is given by

$$q_i = \theta + \eta_i \tag{2.3}$$

where  $\theta$  is a uniformly distributed random variable with support  $[\underline{\theta}, \bar{\theta}]$ , representing the common component of all loss limits. The idiosyncratic component of  $i$ ’s loss limit is given by the random variable  $\eta_i$ , which is uniformly distributed with support  $[-\varepsilon, \varepsilon]$ , and where  $\eta_i$  and  $\eta_j$  for  $i \neq j$  are independent, and  $\eta_i$  is independent of  $\theta$ . Crucially, trader  $i$  knows only of his own loss limit  $q_i$ . He must infer the loss limits of the other traders, based on his knowledge of the joint distribution of  $\{q_j\}$ , and his own loss limit  $q_i$ . The joint density of loss limits described above follows the construction used in the currency attack model of Morris and Shin (1998). We abstract from the issues of public and private signals when the prior density is not uniform (see Morris and Shin, 2003, sect. 3, for details).

## 2.2. EXECUTION OF SELL ORDERS

The trading at date 1 takes place by matching the sales of the risk-neutral traders with the limit buy orders posted by the market-making sector. However, the sequence in which the sell orders are executed is not under the control of the sellers.

We will assume that if the aggregate sale of the asset by the risk-neutral traders is  $s$ , then a seller's place in the queue for execution is uniformly distributed in the interval  $[0, s]$ . Thus the expected price at which trader  $i$ 's sell order is executed is given by

$$v - \frac{1}{2}cs \quad (2.4)$$

and depends on the aggregate sale  $s$ . This feature of our model captures two ingredients. The first is the idea that the price received by a seller depends on the amount sold by other traders. When there is a flood of sell orders (large  $s$ ), then the sale price that can be expected is low. The second ingredient is the departure from the assumption that the transaction price is known with certainty when a trader decides to sell. Even though traders may have a good indication of the price that they can expect by selling (say, through indicative prices), the actual execution price cannot be guaranteed, and will depend on the overall selling pressure in the market. This second feature – the uncertainty of transactions price – is an important feature of a market under stress, and is emphasized by many practitioners (see for instance, Kaufman, 2000, pp. 79–80; Taleb, 1997, pp. 68–69).

The payoff to a seller now depends on whether the execution price is high enough as not to breach the loss limit. Let us denote by  $\hat{s}_i$  the largest value of aggregate sales  $s$  that guarantees that trader  $i$  can execute his sell order without breaching the loss limit. That is,  $\hat{s}_i$  is defined in terms of the equation:

$$q_i = v - c\hat{s}_i \quad (2.5)$$

where the expression on the right hand side is the lowest possible price received by a seller when the aggregate sale is  $\hat{s}_i$ . Thus, whenever  $s \leq \hat{s}_i$ , trader  $i$ 's expected payoff to selling is given by (2.4). However, when  $s > \hat{s}_i$ , there is a positive probability that the loss limit is breached, which leads to the bad payoff of 0. When  $s > \hat{s}_i$ , there is probability  $\hat{s}_i/s$  that trader  $i$ 's order will be executed at a price above his loss limit. Conditional on execution, trader  $i$  can expect to sell at price  $v - \frac{1}{2}c\hat{s}_i$ . With probability  $(s - \hat{s}_i)/s$ , trader  $i$ 's order will be executed at a price below his loss limit. In this case, he is dismissed, resulting in a payoff of zero. Thus, overall, trader  $i$ 's expected payoff to selling is

$$\frac{\hat{s}_i}{s} \left( v - \frac{1}{2}c\hat{s}_i \right) \quad (2.6)$$

If trader  $i$  decides to hold on to the asset, then the payoff is given by the liquidation value of the asset at date 2 if the loss limit is not breached, and 0 if it is breached. Thus, the expected payoff to trader  $i$  of holding the asset, as a function of aggregate sales  $s$ , is

$$u(s) = \begin{cases} v & \text{if } s \leq \hat{s}_i \\ 0 & \text{if } s > \hat{s}_i \end{cases} \quad (2.7)$$

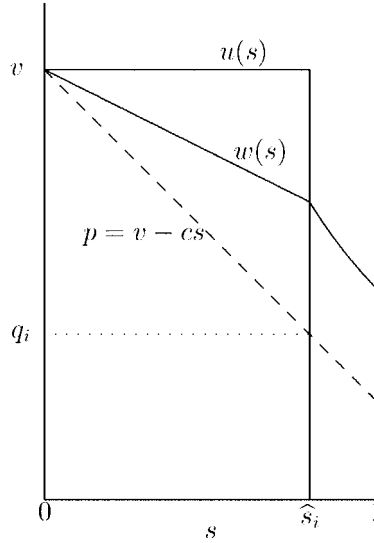


Figure 1.  $u(s)$  is the expected payoff to holding the asset when aggregate sale is  $s$ .  $w(s)$  is the expected payoff to selling the asset when aggregate sale is  $s$ .

Bringing together (2.4) and (2.6), we can write the expected payoff of trader  $i$  from selling the asset as

$$w(s) = \begin{cases} v - \frac{1}{2}cs & \text{if } s \leq \hat{s}_i \\ \frac{\hat{s}_i}{s} (v - \frac{1}{2}c\hat{s}_i) & \text{if } s > \hat{s}_i \end{cases} \quad (2.8)$$

The payoffs are depicted in Figure 1. Holding the asset does better when  $s < \hat{s}_i$ , but selling the asset does better when  $s > \hat{s}_i$ . The trader's optimal action depends on the density over  $s$ . We now solve for equilibrium in this trading game.

### 3. Equilibrium

At date 1, it is common knowledge that the liquidation value at date 2 has mean  $v$  and variance  $\sigma^2$ . Each trader decides whether to sell or hold the asset on the basis of  $v$  and his own loss limit. Trader  $i$ 's strategy is a function

$$(v, q_i) \mapsto \{\text{hold, sell}\}$$

that maps realizations of  $v$  and  $q_i$  to a trading decision. When  $v$  is either very high or very low, trader  $i$  has a dominant action. When  $v$  is very high relative to  $q_i$  (so that the realization of  $v$  is considerably higher than the loss limit for  $i$ ), trader  $i$  will prefer to hold. In particular, since the period 1 price  $p$  cannot fall below  $v - c$ , trader  $i$ 's dominant action is to hold when:

$$v \geq q_i + c \quad (3.1)$$

This is because the loss limit for trader  $i$  will not be breached even if all other traders sell. Conversely, when  $v$  is so low that

$$v < q_i \tag{3.2}$$

then the loss limit is breached even if all other traders hold. Given our assumption that the traders prefer to maximize the value of his trading book conditional on being dismissed, selling is the dominant action when  $v < q_i$ . However, for intermediate values of  $v$  where

$$q_i \leq v < q_i + c \tag{3.3}$$

trader  $i$ 's optimal action depends on the incidence of selling by other traders. If trader  $i$  believes that others are selling, he will sell also. If, however, the others are not selling, then he will hold. If the loss limits were common knowledge, then such interdependence of actions would lead to multiple equilibria, and an indeterminacy in the predicted outcome. When the loss limits are not common knowledge (as in our case), we can eliminate the multiplicity of equilibria through global game techniques.

In particular, we will solve for the unique equilibrium in threshold strategies in which trader  $i$  has the threshold  $v^*(q_i)$  for  $v$  that depends on his own loss limit  $q_i$  such that the equilibrium strategy is given by

$$(v, q_i) \mapsto \begin{cases} \text{hold} & \text{if } v \geq v^*(q_i) \\ \text{sell} & \text{if } v < v^*(q_i) \end{cases} \tag{3.4}$$

In other words,  $v^*(q_i)$  is the trigger level of  $v$  for trader  $i$  such that he sells if and only if  $v$  falls below this critical level. We will show that there is precisely one equilibrium of this kind, and proceed to solve for it by solving for the trigger points  $\{v^*(q_i)\}$ . Our claim can be summarized in terms of the following theorem.

**THEOREM 1.** There is an equilibrium in threshold strategies where the threshold  $v^*(q_i)$  for trader  $i$  is given by the unique value of  $v$  that solves

$$v - q_i = c \exp \left\{ \frac{q_i - v}{2(v + q_i)} \right\} \tag{3.5}$$

There is no other threshold equilibrium.

In a plot with  $v$  on the horizontal axis and  $v - q_i$  on the vertical axis, the left hand side of (3.5) is increasing and passes through  $(q_i, 0)$ , while the right hand side is decreasing in  $v$  and passes through  $(q_i, c)$ . Hence, there is a unique solution to (3.5). At this solution, we must have  $v - q_i > 0$ , so that the trigger point  $v^*(q_i)$  is strictly above the loss limit  $q_i$ . Traders adopt a pre-emptive selling strategy in

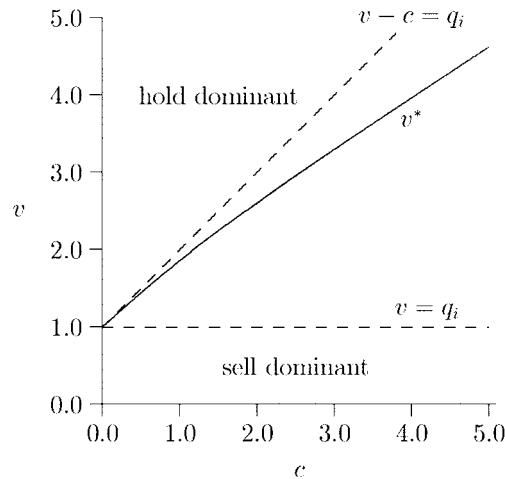


Figure 2. Threshold  $v^*$  as a function of  $c$ , when  $q_i = 1$ . Above the diagonal dotted line, holding the asset is dominant. Below the horizontal dotted line, selling is dominant.

which the trigger level leaves a “margin for prudence”. The intuition here is that a trader anticipates the negative consequences of *other* traders selling. Other traders’ pre-emptive selling strategy must be met by a pre-emptive selling strategy on my part. In equilibrium, every trader adopts an aggressive, pre-emptive selling strategy because others do so. If the traders had long decision horizons, they could ignore the short-term fluctuations in price and hold the asset for its fundamental value. However, traders subject to a loss limit have a short decision horizon. Even though the fundamentals are good, short term price fluctuations can cost him his job. Thus, loss limits inevitably shorten the decision horizon of the traders. The fact that there is a pre-emptive equilibrium of this kind is perhaps not so remarkable. However, what is of interest is the fact that there is no other threshold equilibrium. In particular, the “nice” strategy in which the traders disarm by collectively lowering their threshold points  $v^*(q_i)$  down to their loss limits  $q_i$  cannot figure in any equilibrium behaviour.

Figure 2 plots  $v^*$  as a function of the parameter  $c$  as given by (3.5), while fixing  $q_i = 1$ . Recall that  $c = \gamma\sigma^2$ , where  $\gamma$  is the coefficient of absolute risk aversion. We can see that the critical value  $v^*$  can be substantially higher than the loss limit (given by 1). When  $v$  is very high, so that  $v - c > q_i$ , holding the asset is the dominant action. This dominance region is the area above the upward sloping dashed line in Figure 2. Conversely when  $v < q_i$ , the dominant action is to sell, and this area is indicated as the region below the horizontal dashed line. The large “wedge” between these two dominance regions is the region in which the outcome depends on the resolution of the strategic trading game between the traders. The equilibrium trigger point  $v^*$  bisects this wedge, and determines whether trader  $i$  holds or sells. The solid line plots the equilibrium trigger point given by the solution to (3.5).

Technically, the global game analysed here does not conform to the canonical case discussed in Morris and Shin (2003) in which the payoffs satisfy strategic complementarity, and uniqueness can be proved by the iterated deletion of dominated strategies. In our game, the payoff difference between holding and selling is not a monotonic function of  $s$ . We can see this best from Figure 1. The payoff difference rises initially, but then drops discontinuously, and then rises thereafter, much like the bank run game of Goldstein and Pauzner (2000). Although in this paper we do not address the existence of equilibrium involving strategies other than threshold strategies, it would be possible to prove a tighter result as in Goldstein and Pauzner (2000) to show that our equilibrium is the unique equilibrium. Our argument for the uniqueness of the threshold equilibrium rests on the interaction between strategic uncertainty (uncertainty concerning the actions of other traders) and fundamental uncertainty (uncertainty concerning the fundamentals). Irrespective of the severity of fundamental uncertainty, the strategic uncertainty persists in equilibrium, and the pre-emptive action of the traders reflects the optimal response to strategic uncertainty. Our solution method below will bring this feature out explicitly.

### 3.1. STRATEGIC UNCERTAINTY

The payoff difference between holding the asset and selling the asset when aggregate sales are  $s$  is given by  $u(s) - w(s)$ . The expected payoff advantage of holding the asset over selling it is given by

$$\int_0^1 f(s|v, q_i) [u(s) - w(s)] ds$$

where  $f(s|v, q_i)$  is the density over the equilibrium value of  $s$  (the proportion of traders who sell) conditional on  $v$  and trader  $i$ 's own loss limit  $q_i$ . Trader  $i$  will hold if the integral is positive, and sell if it is negative. Thus, a direct way to solve for our equilibrium is to solve for the density  $f(s|v, q_i)$ .

It is convenient to view the trader's threshold strategy as the choice of a threshold for  $q_i$  as a function of  $v$ . Thus, let us fix  $v$  and suppose that all traders follow the threshold strategy around  $q^*$ , so that trader  $i$  sells if  $q_i > q^*$  and holds if  $q_i \leq q^*$ . Suppose that trader  $i$ 's loss limit  $q_i$  happens to be exactly  $q^*$ . We will derive trader  $i$ 's subjective density over the aggregate sales  $s$ . Since the traders have unit measure, aggregate sales  $s$  is given by the proportion of traders who sell. From trader  $i$ 's point of view,  $s$  is a random variable with support on the unit interval  $[0, 1]$ . The cumulative distribution function over  $s$  viewed from trader  $i$ 's viewpoint can be obtained from the answer to the following question.

“My loss limit is  $q^*$ . What is the probability that  $s$  is less than  $z$ ?” (Q)

The answer to this question will yield  $F(z|q^*)$  – the probability that the proportion of traders who sell is at most  $z$ , conditional on  $q_i = q^*$ . Since all traders are

hypothesized to be using the threshold strategy around  $q^*$ , the proportion of traders who sell is given by the proportion of traders whose loss limits have realizations to the right of  $q^*$ . When the common element of the loss limits is  $\theta$ , the individual loss limits are distributed uniformly over the interval  $[\theta - \varepsilon, \theta + \varepsilon]$ . The traders who sell are those whose loss limits are above  $q^*$ . Hence,

$$s = \frac{\theta + \varepsilon - q^*}{2\varepsilon}.$$

When do we have  $s < z$ ? This happens when  $\theta$  is low enough, so that the area under the density to the right of  $q^*$  is squeezed to a size below  $z$ . There is a value of  $\theta$  at which  $s$  is precisely equal to  $z$ . This is when  $\theta = \theta^*$ , where

$$\theta^* = q^* - \varepsilon + 2\varepsilon z.$$

We have  $s < z$  if and only if  $\theta < \theta^*$ . Thus, we can answer question (Q) by finding the posterior probability that  $\theta < \theta^*$ .

For this, we must turn to trader  $i$ 's posterior density over  $\theta$  conditional on his loss limit being  $q^*$ . This posterior density is uniform over the interval  $[q^* - \varepsilon, q^* + \varepsilon]$ . This is because the ex ante distribution over  $\theta$  is uniform and the idiosyncratic element of the loss limit is uniformly distributed around  $\theta$ . The probability that  $\theta < \theta^*$  is then the area under the posterior density over  $\theta$  to the left of  $\theta^*$ . This is,

$$\frac{\theta^* - (q^* - \varepsilon)}{2\varepsilon} = \frac{q^* - \varepsilon + 2\varepsilon z - (q^* - \varepsilon)}{2\varepsilon} = z \quad (3.6)$$

In other words, the probability that  $s < z$  conditional on loss limit  $q^*$  is exactly  $z$ . The cumulative distribution function  $F(z|q^*)$  is the identity function:

$$F(z|q^*) = z \quad (3.7)$$

The density over  $s$  is then obtained by differentiation.

$$f(s|q^*) = 1 \quad \text{for all } s \quad (3.8)$$

The density over  $s$  is uniform. The noteworthy feature of this result is that the constant  $\varepsilon$  does not enter into the expression for the density over  $s$ . No matter how small or large is the dispersion of loss limits,  $s$  has the uniform density over the unit interval  $[0, 1]$ . In the limit as  $\varepsilon \rightarrow 0$ , every trader's loss limit converges to  $\theta$ . Thus, fundamental uncertainty disappears. Everyone's loss limit converges to the common element  $\theta$ , and everyone knows this fact. And yet, even as fundamental uncertainty disappears, the strategic uncertainty is unchanged. The conclusion that the strategic uncertainty is uniform is quite a general result – it does not depend on the uniform prior and uniform noise. See Morris and Shin (2003, sect. 2) for a full discussion.

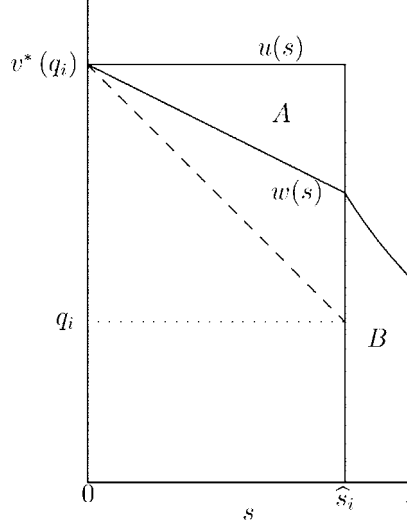


Figure 3. Expected Payoffs in Equilibrium. The threshold point  $v^*$  must be such that the area  $A$  is equal to the area  $B$ .

### 3.2. SOLVING FOR EQUILIBRIUM THRESHOLD

Having found the conditional density over  $s$  at the threshold point  $q^*$ , we can now return to the payoffs of the game. We noted earlier that the expected payoff advantage to holding the asset is given by

$$\int_0^1 f(s|v, q_i) [u(s) - w(s)] ds.$$

At the threshold point  $q^*$ , we have just shown that the density  $f(s|v, q_i)$  is uniform. In addition, the trader is indifferent between holding and selling. Thus, at the threshold point, we have

$$\int_0^1 [u(s) - w(s)] ds = 0.$$

From this equation, we can solve for the threshold point. Figure 3 illustrates the argument. The integral of the payoff difference with respect to a uniform density over  $s$  must be equal to zero. This means that the area labelled  $A$  in Figure 3 must be equal to the area labelled  $B$ .

Substituting in the expressions for (2.7) and (2.8), and noting that  $\hat{s} = (v - q^*)/c$ , we have

$$\frac{1}{2}c \int_0^{\frac{v-q^*}{c}} s ds = \frac{(v - q^*)(v + q^*)}{2cs} \int_{\frac{v-q^*}{c}}^1 \frac{1}{s} ds$$

which simplifies to

$$v - q^* = 2(v + q^*) \log \frac{c}{v - q^*}.$$

Re-arranging this equation gives (3.5) of Theorem 1. There is a unique solution to this equation as already noted, where  $v > q^*$ .

So far, we have shown that if all traders follow the threshold strategy around  $q^*$ , then a trader is indifferent between holding and selling given the threshold loss limit  $q^*$ . We must show that if  $q_i > q^*$ , trader  $i$  prefers to sell, and if  $q_i < q^*$ , trader  $i$  prefers to hold. This step of the argument is presented in the appendix.

#### 4. Discussion

Liquidity black holes are associated with a sharp V-shaped price path for prices. The price at date 1 is given by  $v - cs$ , while the expected value of the asset at date 2 is  $v$ . Thus, the expected return from date 1 to date 2 is given by  $\frac{v}{v - cs}$ . In the limiting case, where the loss limits are perfectly correlated across traders,  $s$  takes the value 1 below  $v^*$ , and takes the value 0 above  $v^*$ . Thus, when there is a liquidity black hole at date 1, the expected return is

$$\frac{v}{v - c}$$

which is strictly larger than the actuarially fair rate of 1 for risk-neutral traders. The larger is  $c$ , the greater is the likely bounce in price. The parameter  $c$  is given by  $c = \gamma\sigma^2$ , where  $\gamma$  is the coefficient of absolute risk aversion and  $\sigma^2$  is the variance of the fundamentals. Since  $c$  gives the slope of the residual demand curve facing the active traders, we can interpret  $c$  as representing the degree of illiquidity of the market. The larger is  $c$ , the smaller is the capacity to absorb the selling pressure from the active traders. Thus, when a liquidity black hole comes into existence, a large  $c$  is associated with a sharper decline in prices, and a commensurate bounce back in prices in the final period.

Another implication of our model is that the trading volume at the time of the liquidity black hole and its aftermath will be considerable. When the market strikes the liquidity black hole, the whole of the asset holding in the risky asset changes hands from the risk-neutral short horizon traders to the risk-averse market making sector. Although we have not modelled the dynamics, we could envisage that immediately afterwards, once the loss limits have been adjusted down given the new price, there will be an immediate reversal of the trades in which the risky asset ends up back in the hands of the risk neutral traders once more. The large trading volume that is generated by these reversals will be associated with the sharp V-shaped price dynamics already noted. The association between the V-shaped pattern in prices and the large trading volume is consistent with the evidence found in Campbell et al. (1993) and Pastor and Stambaugh (2002).

Traders who are aware of their environment take account of limited liquidity in the market. The equilibrium strategies of the traders therefore also take account of the degree of illiquidity of the market. The solution for the threshold point (3.5) shows that when  $c$  increases, the gap between  $v^*$  and  $q_i$  increases also, as shown in Figure 2. In other words, when  $c$  is large, a trader's trigger point  $v^*$  is much higher than his true loss limit  $q_i$ . The trader bails out at a much higher price than his loss limit because he is apprehensive about the effect of other traders bailing out. Just as in the run outcome in a bank run game, the traders in the illiquid market bail out more aggressively when they fear the bailing out of other traders. Since in our model the efficient outcome is for the risk-neutral traders to hold the risky asset, the increase in  $c$  results in a greater welfare loss, *ex ante*.

This last point raises some thorny questions for regulatory policy. While the trigger-happy behaviour of the individual traders is optimal from the point of view of that trader alone, the resulting equilibrium is socially inefficient. In particular when the loss limit of one trader is raised, this has repercussions beyond that individual. For other traders in the market, the raising of the loss limit by one trader imposes an unwelcome negative externality in the form of a more fragile interim price. The natural response of the other traders would be to raise their own loss limits to match. The analogy here is with an arms race.

More generally, when the endogenous nature of price fluctuations is taken into account, the regulatory response to market risk may take on quite a different flavour from the orthodox approach using value at risk using historical prices. Danielsson et al. (2002) and Danielsson and Shin (2003) explore these issues further. These themes are addressed in a comprehensive way by Borio (2003), who argues in favour of a "macro-prudential" approach to financial regulation.

## 5. Appendix

In this appendix, we complete the argument for Theorem 1 by showing that if  $q_i > q^*$ , trader  $i$  prefers to sell, and if  $q_i < q^*$ , trader  $i$  prefers to hold. For this step of the argument, we again appeal to the conditional density over  $s$ . Let us consider a variant of question (Q) for a trader whose loss limit exceeds the threshold point  $q^*$ . Thus, consider the following question.

"My loss limit is  $q_i$  and all others use the threshold strategy around  $q^*$ . What is the probability that  $s$  is less than  $z$ ?"

The answer to this question will yield  $F(z|q_i, q^*)$  – the probability that the proportion of traders who sell is at most  $z$ , conditional on  $q_i$  when all others use the threshold strategy around  $q^*$ . When all traders are using the threshold strategy around  $q^*$ , the proportion of traders who sell is given by the proportion of traders whose loss limits have realizations to the right of  $q^*$ . When the common element of loss limits is  $\theta$ , the individual loss limits are distributed uniformly over the

interval  $[\theta - \varepsilon, \theta + \varepsilon]$ . The traders who sell are those whose loss limits are above  $q^*$ . Hence,

$$s = \frac{\theta + \varepsilon - q^*}{2\varepsilon}.$$

When do we have  $s < z$ ? This happens when  $\theta$  is low enough, so that the area under the density to the right of  $q^*$  is squeezed to a size below  $z$ . There is a value of  $\theta$  at which  $s$  is precisely equal to  $z$ . This is when  $\theta = \theta^*$ , where

$$\theta^* = q^* - \varepsilon + 2\varepsilon z.$$

We have  $s < z$  if and only if  $\theta < \theta^*$ . Thus, we can answer the question posed above by finding the posterior probability that  $\theta < \theta^*$ .

For this, we must turn to trader  $i$ 's posterior density over  $\theta$  conditional on his loss limit being  $q_i$ . This posterior density is uniform over the interval  $[q_i - \varepsilon, q_i + \varepsilon]$ , since the ex ante distribution over  $\theta$  is uniform and the idiosyncratic element of the loss limit is uniformly distributed around  $\theta$ . The conditional probability that  $\theta < \theta^*$  is then the area under the posterior density over  $\theta$  to the left of  $\theta^*$ . This is,

$$\frac{\theta^* - (q_i - \varepsilon)}{2\varepsilon} = \frac{q^* - \varepsilon + 2\varepsilon z - (q_i - \varepsilon)}{2\varepsilon} = z + \frac{q^* - q_i}{2\varepsilon}. \quad (4.1)$$

This gives the cumulative distribution function  $F(z|q_i, q^*)$ , which falls under three cases.

$$F(z|q_i, q^*) = \begin{cases} 0 & \text{if } z + \frac{q^* - q_i}{2\varepsilon} < 0 \\ 1 & \text{if } z + \frac{q^* - q_i}{2\varepsilon} > 1 \\ z + \frac{q^* - q_i}{2\varepsilon} & \text{otherwise} \end{cases}$$

Hence, the corresponding density over  $s$  will, in general, have an atom at either  $s = 0$  or  $s = 1$ . Thus, let us consider  $q_i$  where  $q_i > q^*$ . We show that trader  $i$  does strictly better by selling, than by holding. The conditional density over the half-open interval  $s \in [0, 1)$  is given by

$$f(s|q_i, q^*) = \begin{cases} 0 & \text{if } s < \frac{q_i - q^*}{2\varepsilon} \\ 1 & \text{if } s \geq \frac{q_i - q^*}{2\varepsilon} \end{cases}$$

and there is an atom at  $s = 1$  with weight  $\frac{q_i - q^*}{2\varepsilon}$ .

Meanwhile, from (2.7) and (2.8), the expected payoff advantage of holding relative to selling is given by

$$u - w = \begin{cases} \frac{1}{2}cs & \text{if } s \leq \hat{s}_i \\ -\frac{\hat{s}_i}{s} \left( v - \frac{1}{2}c\hat{s}_i \right) & \text{if } s > \hat{s}_i \end{cases}$$

This payoff function satisfies the single-crossing property in that,  $u - w$  is non-negative when  $s \leq \hat{s}_i$ , and is negative when  $s > \hat{s}_i$ . The density  $f(s|q_i, q^*)$  can be obtained from the uniform density by transferring weight from the interval  $\left[0, \frac{q_i - q^*}{2\varepsilon}\right]$  to the atom on point  $s = 1$ . Since  $\int_0^1 (u(s) - w(s)) ds = 0$ , we must have

$$\int_0^1 (u(s) - w(s)) f(s|q_i, q^*) ds < 0.$$

Thus, the trader with loss limit  $q_i > q^*$  strictly prefers to sell. There is an exactly analogous argument to show that the trader with loss limit  $q_i < q^*$  strictly prefers to hold. This completes the argument for Theorem 1.

## References

- Abreu, Dilip and Brunnermeier, Markus (2003) Bubbles and crashes, *Econometrica* **71**, 173–204.
- Acharya, Viral V. and Pedersen, Lasse Heje (2002) Asset pricing with liquidity risk, Working Paper, London Business School and NYU Stern School.
- Allen, Franklin and Gale, Douglas (2002) Financial fragility, Working Paper 01-37, Wharton Financial Institutions Center, University of Pennsylvania, <<http://fic.wharton.upenn.edu/fic/papers/01/0137.pdf>>.
- Bernardo, Antonio and Welch, Ivo (2002) Financial market runs, *Quarterly Journal of Economics*, forthcoming.
- Borio, Claudio (2003) Towards a macroprudential approach to financial supervision and regulation? BIS Working Paper 128, Bank for International Settlements, <<http://www.bis.org/publ/work128>>.
- Bruche, Max (2002) A structural model of corporate bond prices with coordination failure, FMG Discussion Paper 410, LSE.
- Brunnermeier, Markus and Pedersen, Lasse Heje (2002) Predatory trading, Working Paper, Princeton University and NYU Stern School.
- Campbell, John, Grossman, Sanford J. and Wang, Jiang (1993) Trading volume and serial correlation in stock returns, *Quarterly Journal of Economics* **108**, 905–939.
- Cohen, Benjamin and Shin, Hyun Song (2001) Positive feedback trading under stress: Evidence from the US Treasury securities market, unpublished paper, <<http://www.nuff.ox.ac.uk/users/shin/working.htm>>.
- Danielsson, Jon and Payne, Richard (2001) Measuring and explaining liquidity on an electronic limit order book: Evidence from Reuters D2000-2, Working Paper, London School of Economics.
- Danielsson, Jon and Shin, Hyun Song (2002) *Endogenous Risk*, in *Modern Risk Management: A History*, Risk Books, London, <<http://www.nuff.ox.ac.uk/users/shin/policy.htm>>.
- Danielsson, Jon, Shin, Hyun Song, and Zigrand, Jean-Pierre (2002) The impact of risk regulation on price dynamics, *Journal of Banking and Finance*, forthcoming.

- Diamond, Douglas and Dybvig, Philip (1983) Bank runs, deposit insurance, and liquidity, *Journal of Political Economics* **91**, 401–419.
- Diamond, Douglas and Rajan, Raghuram (2000) Liquidity shortages and banking crises, Working Paper, University of Chicago, GSB.
- Evans, Martin D. D. and Lyons, Richard K. (2003) How is macro news transmitted to exchange rates? NBER Working Paper 9433.
- Fleming, Michael J. and Remolona, Eli M. (1999) Price formation and liquidity in the US Treasury market: The response to public information, *Journal of Finance* **54**, 1901–1915.
- Genotte, Gerard and Leland, Hayne (1990) Hedging and crashes, *American Economic Review*, 999–1021.
- Goldstein, Itay and Pauzner, Ady (2000) Demand deposit contracts and the probability of bank runs, Working Paper, Duke University and Tel Aviv University.
- Gorton, Gary and Huang, Lixin (2003) Liquidity and efficiency, Working Paper, Wharton School, University of Pennsylvania.
- Gromb, Denis and Vayanos, Dimitri (2002) Equilibrium and welfare in markets with financially constrained arbitrageurs, *Journal of Financial Economics* **66**, 361–407.
- Grossman, Sanford J. and Miller, Merton H. (1988) Liquidity and market structure, *Journal of Finance* **43**, 617–633.
- Hasbrouck, Joel (2000) Trading fast and slow: Security market events in real time, unpublished manuscript, New York University.
- Holmström, Bengt and Tirole, Jean (2001) LAPM: A liquidity-based asset pricing model, *Journal of Finance* **56**, 1837–1867.
- Kaufman, Henry (2000) *On Money and Markets: A Wall Street Memoir*, McGraw-Hill, New York.
- Kyle, A. S. and Wei, Xiong (2001) Contagion as a wealth effect, *Journal of Finance* **56**, 1401–1440.
- Lustig, Hanno (2001) The market price of aggregate risk and the wealth distribution, Working Paper, Stanford University.
- Morris, Stephen and Shin, Hyun Song (1998) Unique equilibrium in a model of self-fulfilling currency attacks, *American Economic Review* **88**, 587–597.
- Morris, Stephen and Shin, Hyun Song (2003) Global games: Theory and applications, in M. Dewatripont, L. Hansen and S. Turnovsky (eds.), *Advances in Economics and Econometrics, the Eighth World Congress*, Cambridge University Press.
- Osler, Carol (2002) Stop-loss orders and currency cascades in currency markets, Working Paper, Brandeis University.
- Pastor, Lubos and Stambaugh, Robert (2002) Liquidity risk and expected stock returns, *Journal of Political Economy* forthcoming.
- Persaud, Avinash (2001) Liquidity black holes, Working Paper, State Street Bank, <[http://www.statestreet.com/knowledge/research/liquidity\\_black\\_holes.pdf](http://www.statestreet.com/knowledge/research/liquidity_black_holes.pdf)>.
- Plantin, Guillaume (2003) A theory of self-fulfilling liquidity, LSE Financial Markets Group.
- Schnabel, Isabel and Shin, Hyun Song (2002) Foreshadowing LTCM: The crisis of 1763, Working Paper, Mannheim University and London School of Economics, <<http://www.nuff.ox.ac.uk/users/shin/working.htm>>.
- Shleifer, Andrei and Vishny, Robert (1997) The limits to arbitrage, *Journal of Finance* **52**, 35–55.
- Taleb, Nassim (1997) *Dynamic Hedging*, John Wiley, New York.